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# CORRELATING EQUATIONS FOR NATURAL CONVECTION HEAT TRANSFER BETWEEN HORIZONTAL CIRCULAR CYLINDERS

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**Abstract**—Correlating equations for heat transfer by natural convection from horizontal cylinders to a cylindrical enclosure are obtained using a conduction boundary-layer model. The correlation is valid for heat transfer by conduction, laminar flow and turbulent flow. The results approach the correlation for heat transfer from a free horizontal cylinder as the outer cylinder diameter becomes infinite and for quasi-steady heat transfer to fluid within a horizontal cylinder as the inner cylinder diameter approaches zero. Horizontal concentric, eccentric and arrays of cylinders within the outer cylinder are geometries included in the correlation.

## NOMENCLATURE

$A_i$, inner cylinder area;
$D_i$, inner cylinder diameter;
$D_o$, outer cylinder diameter;
$\bar{h}$, mean overall heat-transfer coefficient;
$\bar{h}_i$, mean inner cylinder heat-transfer coefficient, $= Q/\pi D_i l(T_i - \bar{T}_b)$;
$\bar{h}_o$, mean outer cylinder heat-transfer coefficient, $= Q/\pi D_o l(\bar{T}_b - T_o)$;
$k$, thermal conductivity;
$\overline{k_{eq}}$, mean equivalent conductivity;
$l$, length along cylinder;
$L$, gap width, $= R_o - R_i$;
$N$, number of inner cylinders;
$\overline{Nu}_i$, mean Nusselt number for inner cylinder boundary-layer conditions, $= \bar{h}_i D_i/k$;
$\overline{Nu}_o$, mean Nusselt number for outer cylinder boundary-layer conditions, $= \bar{h}_o D_o/k$;
$\overline{Nu}'_{D_i}$, mean Nusselt number for conduction or convection within an enclosure;
$\overline{Nu}'_{D_{i,conv}}$, mean Nusselt number for convection, $= \bar{h}_i D_i/k$, $Q = \bar{h}_i \pi D_i l(T_i - \bar{T}_b)$;
$\overline{Nu}'_{D_{i,conv}}$, mean Nusselt number for convection, $= \bar{h} D_i/k$, $Q = \bar{h}\pi D_i l(T_i - T_o)$;
$\overline{Nu}'_{D_{i,cond}}$, mean Nusselt number for conduction within an enclosure;
$\overline{Nu}_{D_{o,conv}}$, mean Nusselt number for convection within an enclosure, $= \bar{h} D_o/k$, $Q = \bar{h}\pi D_o l(\bar{T}_b - T_o)$;
$Pr$, Prandtl number;
$Q$, rate of total heat flow;
$R_i$, inner cylinder radius;
$R_o$, outer cylinder radius;
$Ra$, Rayleigh number;
$Ra'$, Rayleigh number using $\Delta T = T_i - T_0$;
$T$, temperature.

Greek symbols

$\bar{\delta}_i$, mean thickness of inner cylinder film;
$\bar{\delta}_o$, mean thickness of outer cylinder film;
$\varepsilon$, eccentricity of inner cylinder: distance moved from its concentric position.

Subscripts

$b$, bulk;
$D$, cylinder diameter used as length scale;
$i$, inner cylinder;
$L$, gap width used as length scale;
$o$, outer cylinder.

## INTRODUCTION

NUMEROUS correlations have been proposed for overall heat transfer by natural convection between horizontal concentric cylinders. Kraussold [1] and Bishop [2] found the mean equivalent conductivity to be essentially a function of Rayleigh number based on the gap thickness for $L/D_i < 3$. A better correlation was obtained by Itoh, Fujita, Nishiwaki and Hirata [3] who used $\sqrt{(R_i R_o)}\ln(R_i/R_o)$ as the characteristic length. Raithby and Hollands [4] used a conduction layer model similar to that first proposed by Langmuir as reported by Eckert [5] but without curvature effects. Another conduction model was used by Barelko and Shtessel [6].

Several heat-transfer correlations for natural convection from a single horizontal cylinder have been obtained including those by McAdams [7], Raithby and Hollands [8] and Churchill and Chu [9]. The only correlations developed for all Rayleigh and Prandtl numbers are those in [8] and [9].

The present method of correlating overall heat-transfer results combines conduction solutions, laminar boundary-layer solutions and experimental data. A single correlation is developed to predict heat transfer from a single horizontal cylinder, to a fluid inside a horizontal cylinder under quasi-steady conditions and between concentric and eccentric cylinders. Slight modifications enable the heat-transfer coefficients for more than one inner cylinder within an outer cylinder to be predicted.

## ANALYSIS

Consider two infinitely long horizontal cylinders maintained at different temperatures with the smaller diameter cylinder positioned within the larger one. Heat is transferred through a fluid contained in the space between the cylinders by natural convection. A conduction film is assumed to exist in the fluid near the surface of each cylinder which constitutes the thermal resistance. The central fluid region is assumed to have an average bulk temperature, $\overline{T}_b$. Thermal energy is transported from the inner film to the outer film by convection with no losses. The average heat-transfer coefficient for the inner film determined from

$$Q = \overline{h}_i A_i (T_i - \overline{T}_b) \quad (1)$$

is given by

$$\overline{h}_i = \frac{2k}{D_i \ln\left[\dfrac{D_i + 2\overline{\delta}_i}{D_i}\right]}. \quad (2)$$

This is simply conduction through an annulus with an inner diameter $D_i$, mean thickness $\overline{\delta}_i$ and thermal conductivity $k$. Similarly at the outer cylinder the heat-transfer coefficient based on the area of the outer cylinder is

$$\overline{h}_o = \frac{2k}{D_o \ln\left[\dfrac{D_o}{D_o - 2\overline{\delta}_o}\right]}. \quad (3)$$

Combining the two thermal resistances in series gives an expression for the overall heat-transfer coefficient for the cylinders

$$\overline{h}D_i = \left[\frac{1}{\overline{h}_i D_i} + \frac{1}{\overline{h}_o D_o}\right]^{-1}. \quad (4)$$

The average Nusselt number for convection is found by combining equations (2), (3) and (4)

$$\frac{\overline{h}D_i}{k} = \overline{Nu}_{D_{i\,conv}} = \frac{2}{\ln\left[\dfrac{1 + 2\overline{\delta}_i/D_i}{1 - 2\overline{\delta}_o/D_o}\right]} \quad (5)$$

which can be written

$$\overline{Nu}_{D_{i\,conv}} = \frac{2}{\ln\left[\dfrac{1 + 2/\overline{Nu}_i}{1 - 2/\overline{Nu}_o}\right]}. \quad (6)$$

The significance of the term $\overline{Nu}_i$ becomes clear in the limit as $D_o \to \infty$. This corresponds to heat transfer by natural convection from the outside surface of a single horizontal cylinder. Equation (6) reduces to

$$\overline{Nu}_{D_{i\,conv}} = \frac{2}{\ln[1 + 2/\overline{Nu}_i]}. \quad (7)$$

When the thickness of the boundary layer ($\overline{\delta}_i$) is very small compared to the cylinder diameter ($D_i$) (7) reduces to

$$\overline{Nu}_{D_{i\,conv}} = \overline{Nu}_i. \quad (8)$$

This is the boundary-layer approximation in which curvature effects are neglected.

For laminar flow $\overline{Nu}_i$ can be written as

$$\overline{Nu}_i = 0.518 Ra_{D_i}^{1/4}\left[1 + \left(\frac{0.559}{Pr}\right)^{3/5}\right]^{-5/12} \quad (9)$$

which is in the form used in [9], to correlate laminar boundary-layer heat transfer by natural convection from a horizontal cylinder. The power in the Prandtl number term is here modified to fit the boundary-layer solutions of Chiang and Kaye [10] and Lin and Chao [11] at $Pr = 0.7$ rather than that of Saville and Churchill [12] which is slightly lower than the others. The temperature difference used in the Rayleigh number is $T_i - \overline{T}_b$ and the length scale $D_i$.

No boundary-layer solutions are available for turbulent flow from a single horizontal cylinder. The expression

$$\overline{Nu}_i = 0.1 Ra_{D_i}^{1/3} \quad (10)$$

correlates experimental heat-transfer data for gases and liquids as mentioned by Cess [13] and correlates the turbulent mass-transfer data of Schütz [14] very well.

The two relations for laminar and turbulent flow can be combined using the method of Churchill and Usagi [15]. To give an expression valid for all boundary-layer flow conditions

$$\overline{Nu}_i = \left[\left(0.518 Ra_{D_i}^{1/4}\left[1 + \left(\frac{0.559}{Pr}\right)^{3/5}\right]^{-5/12}\right)^{15} + (0.1 Ra_{D_i}^{1/3})^{15}\right]^{1/15} \quad (11)$$

The exponent 15 was chosen somewhat arbitrarily to fit the experimental data.

Placing this expression for $\overline{Nu}_i$ into (7) incorporates curvature effects and gives a correlation for natural convection heat transfer from a single horizontal cylinder valid at any Rayleigh and Prandtl number.

$$\overline{Nu}_{D_{i\,conv}} = \frac{2}{\ln\left[1 + \dfrac{2}{\left[\left(0.518 Ra_{D_i}^{1/4}\left[1 + \left(\dfrac{0.559}{Pr}\right)^{3/5}\right]^{-5/12}\right)^{15} + (0.1 Ra_{D_i}^{1/3})^{15}\right]^{1/15}}\right]} \quad (12)$$

Results from this expression at $Pr = 0.7$ are listed in Table 1 with numerical values obtained from previous correlations. The present values are very close to those of [8] in particular at small Rayleigh numbers. These results fit the data of Collis and Williams [16] much better than the correlation of [9].

The Nusselt number should approach zero as the Rayleigh number decreases since the heat transfer by conduction from a cylinder to an infinitely large medium is zero. This is not true for the correlation presented in [9] which uses an empirical non-zero constant as the lower limiting value.

Table 1. Comparison of correlations for the overall Nusselt number for natural convection from a free horizontal cylinder to gases ($Pr = 0.7$)

| $\log_{10} Ra_{D_i}$ | $\overline{Nu}_{D_{i_{conv}}}$ | | | | | |
|---|---|---|---|---|---|---|
| | [7] | [23] | [24] | [9] | [8] | Equation (12) |
| 13 | | | | 2276 | 2123 | 2155 |
| 12 | | | | 1069 | 992 | 1001 |
| 11 | | | | 505 | 466 | 465 |
| 10 | | | | 240 | 221 | 216 |
| 9 | 93.3 | 88.3 | | 116 | 107 | 101 |
| 8 | 51.3 | 49.0 | | 56.5 | 53.0 | 47.7 |
| 7 | 28.8 | 27.4 | | 28.2 | 27.3 | 24.1 |
| 6 | 16.2 | 15.5 | | 14.5 | 14.7 | 13.6 |
| 5 | 9.33 | 8.93 | | 7.76 | 8.32 | 8.05 |
| 4 | 5.37 | 5.31 | | 4.37 | 4.97 | 4.92 |
| 3 | 3.16 | 3.31 | | 2.61 | 3.13 | 3.14 |
| 2 | 2.11 | 2.17 | 2.11 | 1.67 | 2.09 | 2.10 |
| 1 | 1.51 | 1.44 | 1.51 | 1.15 | 1.48 | 1.49 |
| 0 | 1.08 | 0.957 | 1.07 | 0.848 | 1.11 | 1.11 |
| −1 | 0.841 | 0.697 | 0.800 | 0.670 | 0.868 | 0.872 |
| −2 | 0.661 | 0.542 | 0.596 | 0.561 | 0.706 | 0.708 |
| −3 | 0.550 | 0.442 | 0.525 | 0.492 | 0.591 | 0.593 |
| −4 | 0.490 | 0.372 | 0.463 | 0.448 | 0.507 | 0.508 |
| −5 | | 0.322 | | 0.419 | 0.443 | 0.444 |
| −6 | | 0.283 | | 0.400 | 0.393 | 0.394 |
| −7 | | 0.252 | | 0.387 | 0.354 | 0.354 |
| −8 | | | | 0.378 | 0.321 | 0.322 |
| −9 | | | | 0.372 | 0.294 | 0.294 |
| −10 | | | | 0.368 | 0.271 | 0.271 |
| −11 | | | | 0.366 | 0.251 | 0.252 |
| −12 | | | | 0.364 | 0.234 | 0.235 |
| −13 | | | | 0.363 | 0.220 | 0.220 |

Taking the limit as the inner cylinder diameter approaches zero in equation (6) gives

$$\overline{Nu}_{D_{i_{conv}}} = \frac{2}{-\ln[1-2/\overline{Nu}_o]} \tag{13}$$

which can be written

$$\overline{Nu}_{D_{o_{conv}}} = \frac{2}{-\ln[1-2/\overline{Nu}_o]} \tag{14}$$

where $\overline{Nu}_{D_{o_{conv}}}$ uses $D_o$ as the reference length in place of $D_i$. As curvature effects disappear this reduces to

$$\overline{Nu}_{D_{o_{conv}}} = \overline{Nu}_o. \tag{15}$$

This corresponds to natural convection boundary-layer heat transfer to a fluid inside a horizontal cylinder under quasi-steady conditions with no curvature effects.

No boundary-layer solutions were found in the literature for this situation although experiments have been performed with gases and liquids. Deaver and Eckert [17] present heat-transfer results at low Rayleigh numbers where conduction and curvature effects are not negligible. Maas and David [18] report a similar study at higher Rayleigh numbers where the liquid results could be correlated by

$$\overline{Nu}_o = 0.587 Ra_{D_o}^{1/4} \tag{16}$$

with the temperature difference in the Rayleigh number $\bar{T}_b - T_o$ and the length scale $D_o$. At small Rayleigh numbers Deaver and Eckert show that $\overline{Nu}_{D_{o_{conv}}}$ in equation (14) becomes that for conduction which is 8.0. Therefore, as the Rayleigh number approaches zero

$$\overline{Nu}_o = \frac{2}{1-e^{-0.25}}. \tag{17}$$

Equations (16) and (17) were combined with equation (14) to fit the data in [17]. This results in

$$\overline{Nu}_o = \left[\left(\frac{2}{1-e^{-0.25}}\right)^{5/3} + (0.587 Ra_{D_o}^{1/4})^{5/3}\right]^{3/5} \tag{18}$$

which is valid for conduction and laminar flow with the exponent 5/3 chosen empirically.

No experiments or boundary-layer solutions were found for turbulent flow although Schmidt [19] found that turbulent natural convection within a sphere could be correlated by

$$\overline{Nu}_o = 0.119 Ra_{Do}^{1/3}. \tag{19}$$

An equation similar to (10) is chosen as being a reasonable estimate for turbulent flow within horizontal cylinders for gases and liquids

$$\overline{Nu}_o = 0.1 Ra_{Do}^{1/3}. \tag{20}$$

Equations (18) and (20) were combined to give an expression for $\overline{Nu}_o$ valid at all Rayleigh numbers

$$\overline{Nu}_o = \left\{\left(\left[\left(\frac{2}{1-e^{-0.25}}\right)^{5/3} + (0.587 Ra_{Do}^{1/4})^{5/3}\right]^{3/5}\right)^{15} + (0.1 Ra_{Do}^{1/3})^{15}\right\}^{1/15}. \tag{21}$$

This is placed in (14) to give a correlation for quasi-steady natural convection heat transfer to a fluid inside a horizontal cylinder valid at any Rayleigh number

$$\overline{Nu}_{D_{o\,conv}} = \frac{2}{-\ln\left[1 - \dfrac{2}{\left\{\left(\left[\left(\dfrac{2}{1-e^{-0.25}}\right)^{5/3} + (0.587 Ra_{Do}^{1/4})^{5/3}\right]^{3/5}\right)^{15} + (0.1 Ra_{Do}^{1/3})^{15}\right\}^{1/15}}\right]}. \tag{22}$$

The values of the Nusselt number obtained from this expression are plotted on Fig. 1. The experimental points of Deaver and Eckert are correlated very well. The correlation of Maas and David becomes the limiting case for laminar flow at large Rayleigh numbers. All the experiments used in obtaining the present correlation were performed using moderate Prandtl number liquids. This correlation is not expected to give valid results for low Prandtl number fluids.

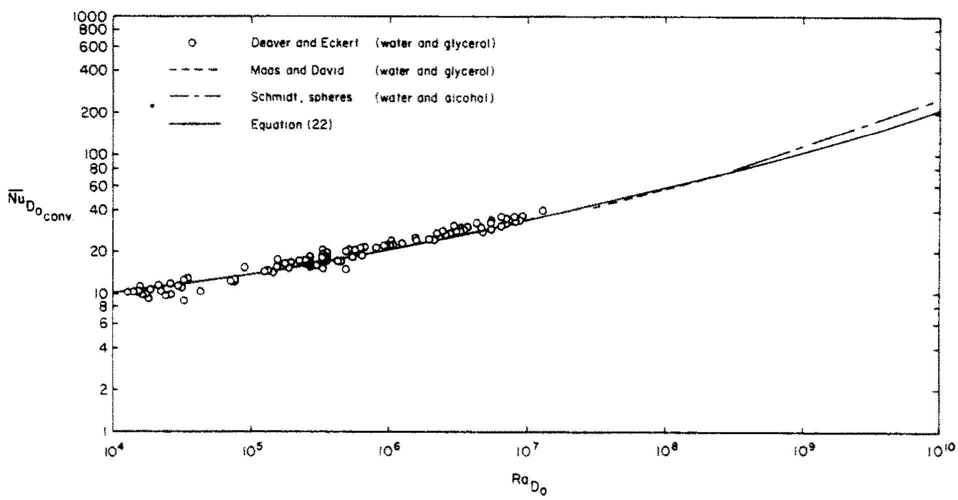


FIG. 1. Comparison of correlating equation with experimental results for quasi-steady natural convection to liquids in a horizontal cylinder.

The expressions for $\overline{Nu}_i$ and $\overline{Nu}_o$ can be combined with equation (6) to give a relation for the overall Nusselt number for heat transfer by natural convection between an inner and outer horizontal cylinder. A Prandtl number variation was incorporated into the correlation for $\overline{Nu}_o$ to fit numerical results [29] for the horizontal concentric cylinder configuration at $Ra_L = 10^4$. $L/D_i = 0.8$ and $0.01 \leqslant Pr \leqslant 1000$ to within 2%. The resulting correlation becomes

$$\overline{Nu}'_{D_{i\,conv}} = \frac{2}{\ln\left[\dfrac{1 + \dfrac{2}{\left[\left(0.518 Ra_{D_i}^{1/4}\left[1+\left(\dfrac{0.559}{Pr}\right)^{3/5}\right]^{-5/12}\right)^{15} + (0.1 Ra_{D_i}^{1/3})^{15}\right]^{1/15}}}{1 - \dfrac{2}{\left\{\left(\left[\left(\dfrac{2}{1-e^{-0.25}}\right)^{5/3} + (0.587 G Ra_{Do}^{1/4})^{5/3}\right]^{3/5}\right)^{15} + (0.1 Ra_{Do}^{1/3})^{15}\right\}^{1/15}}}\right]} \tag{23a}$$

$$G = \left[\left(1 + \frac{0.6}{Pr^{0.7}}\right)^{-5} + (0.4 + 2.6 Pr^{0.7})^{-5}\right]^{-1/5} \tag{23b}$$

The relation for $G$ can be used in equation (22) also.

This cannot be evaluated until $\bar{T}_b$ is known. However, it is easily found by equating the heat transfer at each cylinder which results in

$$\frac{\bar{T}_b - T_o}{T_i - \bar{T}_b} = \frac{\overline{Nu}_{D_{i\,conv}}}{\overline{Nu}_{D_{o\,conv}}} \quad (24)$$

with $\overline{Nu}_{D_{i\,conv}}$ and $\overline{Nu}_{D_{o\,conv}}$ given by equations (12) and (22) respectively. In practice four or five iterations are required to establish $\bar{T}_b$ from an initial guess value so that (23) can be evaluated.

At small Rayleigh numbers the heat transfer between the cylinders is by conduction. The presence of the cylinders limits the size of $\bar{\delta}_i$ and $\bar{\delta}_o$ which was not considered previously. Therefore, the Nusselt number for conduction becomes the lower limiting value of (23). For conduction between two cylinders

$$\overline{Nu}'_{D_{i\,cond}} = \frac{2}{\cosh^{-1}[(D_i^2 + D_o^2 - 4\varepsilon^2)/2D_iD_o]} \quad (25)$$

with $\varepsilon$ the distance the inner cylinder is moved from its concentric position.

The value of the overall Nusselt number valid at any Rayleigh number is found by combining equations (23) and (25)

$$\overline{Nu}'_{D_i} = [(\overline{Nu}'_{D_{i\,cond}})^{15} + (\overline{Nu}'_{D_{i\,conv}})^{15}]^{1/15} \quad (26)$$

with the exponent 15 chosen to fit experimental data. The overall equivalent conductivity is defined as

$$\overline{k_{eq}} = \frac{\overline{Nu}'_{D_i}}{\overline{Nu}'_{D_{i\,cond}}} \quad (27)$$

Equations (23) through (27) should correlate heat-transfer results for natural convection between two horizontal concentric or eccentric cylinders.

## RESULTS AND DISCUSSION

When the eccentricity is zero, results for concentric cylinders should be obtained. Experimental results for this configuration obtained with gases and liquids are shown on Fig. 2 as are the curves calculated with $D_o/D_i = 2$ and 3 at $Pr = 0.7$. The data in both the laminar and turbulent regions agree well with the present correlation. The two curves do not differ substantially. However, curves for $D_o/D_i > 10$ have smaller slopes and become turbulent at larger values of $Ra_L$ than those shown on Fig. 2. The change in slope was found experimentally by Grigull and Hauf [27] who incorporated it into their heat-transfer correlation.

As the diameter of the outer cylinder is increased, the heat transfer approaches that of a single horizontal cylinder. Curves for various $D_o/D_i$ ratios are given on Fig. 3 at $Pr = 100$. This Prandtl number is chosen since the results are virtually independent of the Prandtl number correlations used in equation (23). The temperature difference used in the Rayleigh number is $T_i - T_o$. The curve for $D_o/D_i = \infty$ corresponds to a free horizontal cylinder. The heat-transfer coefficient for concentric cylinders is lower than that for a free cylinder except when conduction is predominant. The results approach the free cylinder limit monotonically, not in an oscillatory manner predicted in [6]. There are no discontinuities in the results as in the correlation presented by Powe [22]. To have a heat-transfer coefficient within 5% of that for a free cylinder requires $D_o/D_i > 360$ at $Ra_{D_i} = 10^7$ and $D_o/D_i > 700$ at $Ra_{D_i} = 10^{-1}$.

Only a limited amount of data has been obtained for natural convection between horizontal eccentric cylinders. Zagromov and Lyalikov [20] show that the heat transfer is essentially the same as with a concentric geometry until $\varepsilon/L \approx 1$. Figure 4 shows experimental data obtained using air and numerical solutions for eccentric cylinders at $Pr = 0.7$, from [21]. These correspond to a heated inner cylinder moved below center with $\varepsilon/L = 0.325$ and $D_o/D_i = 2.6$. The present correlation obtained with $Pr = 0.7$ agrees fairly well with the experiments and does not deviate more than 10% from the numerical solutions.

More than one inner cylinder inside a cylindrical

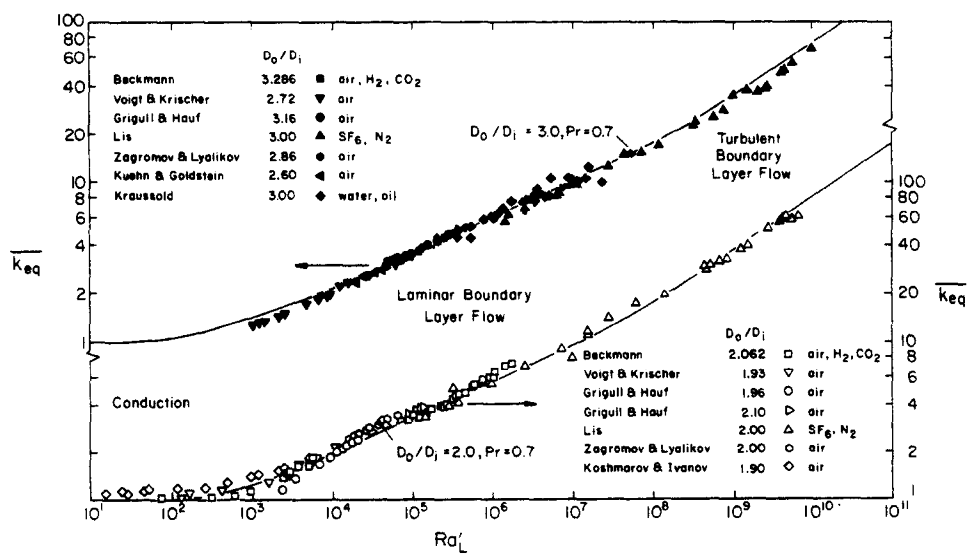


FIG. 2. Comparison of correlating equations with experimental results for natural convection between horizontal concentric cylinders.

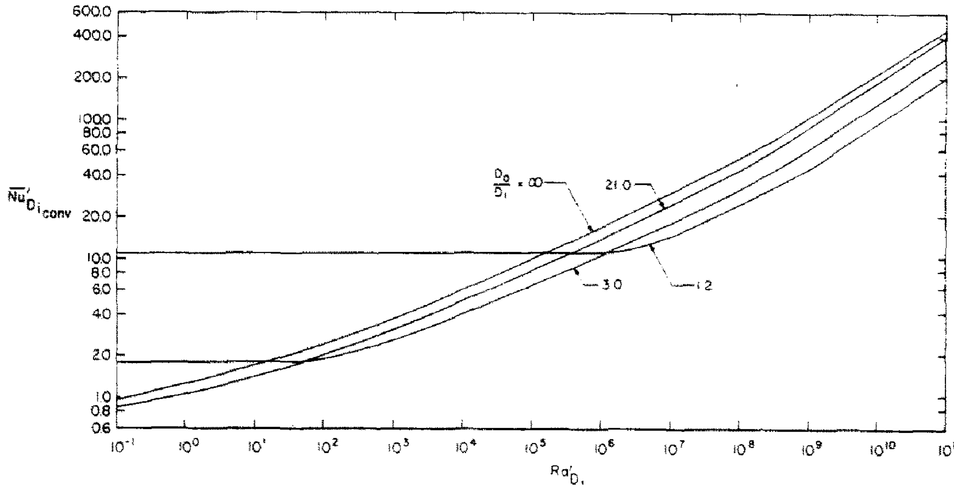


FIG. 3. Effect of $D_o/D_i$ on the heat transfer between horizontal concentric cylinders at $Pr = 100$.

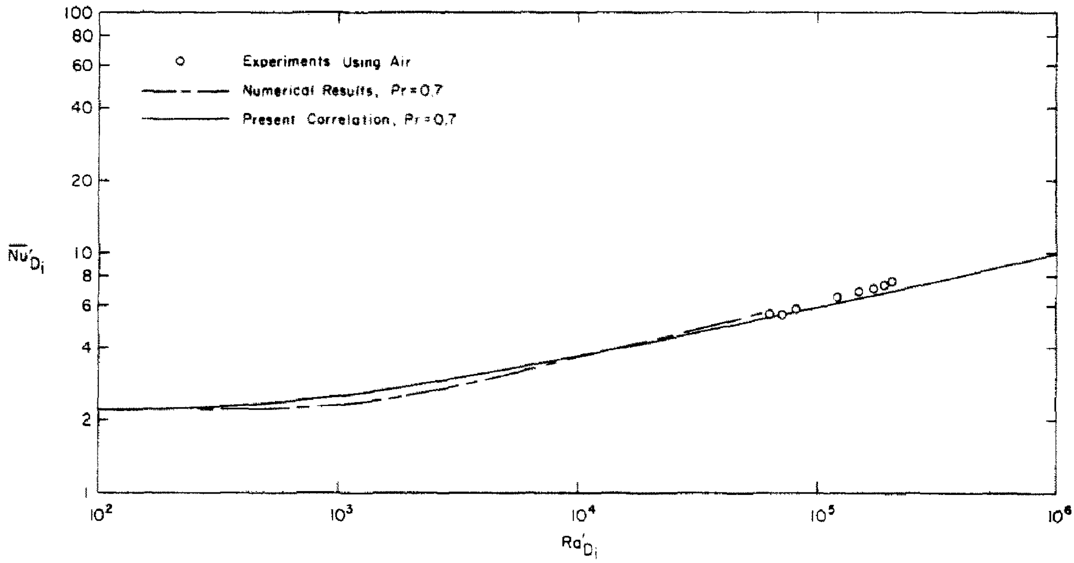


FIG. 4. Comparison of correlating equations and experimental and numerical results for natural convection between horizontal eccentric cylinders, $D_o/D_i = 2.6$, $\varepsilon/L = 0.325$.

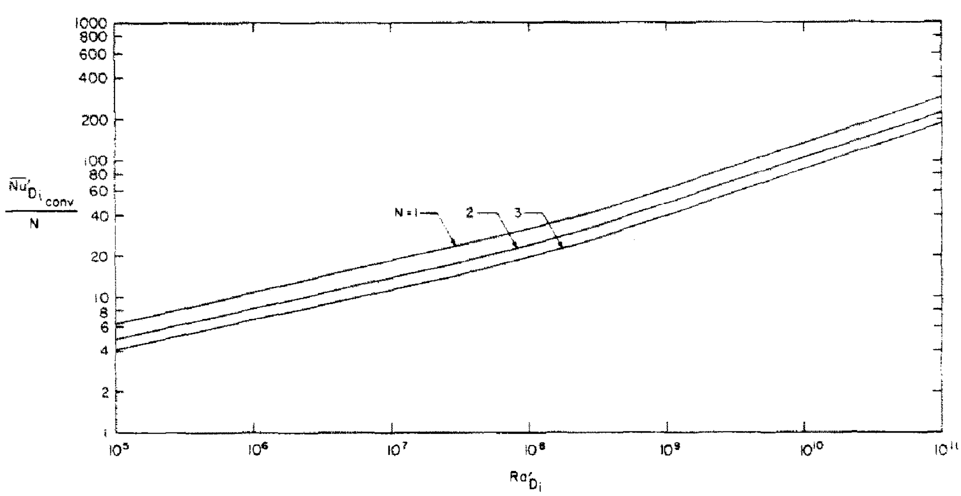


FIG. 5. Mean Nusselt number for natural convection from inner cylinders to a horizontal cylindrical enclosure, $D_o/D_i = 3.0$, $Pr = 100$.

enclosure can be treated by summing the heat transfer from each inner cylinder assuming $\bar{T}_b$ is the same for each. This is not expected to give realistic values when a large number of inner cylinders are used since the bulk temperature will vary considerably at different locations. If the inner cylinders have the same diameter and are maintained at the same temperature the overall Nusselt number for convection becomes

$$\overline{Nu}'_{D_{i_{conv}}} = \frac{2N}{\ln[1+2/\overline{Nu}_i]-N\ln[1-2/\overline{Nu}_o]} \quad (28)$$

with $\overline{Nu}_i$ and $\overline{Nu}_o$ obtained from (11) and (21) respectively and $N$ set equal to the number of inner cylinders. The heat-transfer coefficient in $\overline{Nu}_{D_{i_{conv}}}$ is based on the surface area of one of the inner cylinders. $\bar{T}_b$ is computed from similar to (22) from

$$\frac{\bar{T}_b - T_o}{T_i - \bar{T}_b} = \frac{N\overline{Nu}_{D_{i_{conv}}}}{\overline{Nu}_{D_{o_{conv}}}}. \quad (29)$$

The value for conduction must be found and substituted for (25) to obtain heat-transfer results at small Rayleigh numbers. Figure 5 shows the results for 1, 2 and 3 inner cylinders inside a cylindrical enclosure. The diameter ratio for each case is $D_o/D_i = 3$ at a Prandtl number of 100. The heat-transfer coefficient for the inner cylinders decreases as more cylinders are added. That for three inner cylinders is approximately two-thirds of the value for a single inner cylinder with the same temperature difference $T_i - T_o$ and Rayleigh number based on $D_i$. The mean bulk temperature increases with the number of inner cylinders. This is expected since more heat must be transferred to the outer cylinder across the temperature difference $\bar{T}_b - T_o$. No other heat-transfer results could be found for this configuration although three or four cylinders within a horizontal cylindrical enclosure are proposed designs for cryogenic and compressed-gas-insulated electric power cables [31, 32].

## SUMMARY

A conduction boundary-layer model has been developed to correlate existing overall heat-transfer results for natural convection between isothermal horizontal concentric and eccentric cylinders. The present result approaches previous correlations for a free horizontal cylinder for laminar and turbulent flow in the limit as $D_o \to \infty$. Experimental data for quasi-steady heat transfer within a horizontal cylinder is also correlated in the limit as $D_i \to 0$. Heat-transfer coefficients for more than one inner cylinder within a cylindrical enclosure are also predicted.

The present correlation has been developed assuming isothermal cylinders. Any thermal boundary condition may be imposed providing corresponding boundary-layer solutions or experiments are used to define $\overline{Nu}_i$ and $\overline{Nu}_o$.

The correlation gives valid results for a number of geometries with little or no discontinuity between them. This is preferred to developing a separate correlation for every geometry encountered. Fewer correlations are required and heat-transfer coefficients can be obtained for geometries that have not been investigated previously.

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## EQUATIONS DE CORRELATION POUR LE TRANSFERT DE CHALEUR PAR CONVECTION NATURELLE ENTRE CYLINDRES CIRCULAIRES HORIZONTAUX

**Résumé**—Des équations de corrélation pour le transfert de chaleur par convection naturelle autour de cylindres horizontaux à l'intérieur d'une enceinte cylindrique sont obtenues sur la base d'un modèle de couche limite de conduction. La relation est valable pour le transfert de chaleur par conduction, en régimes laminaire et turbulent. Les résultats tendent vers l'équation du transfert de chaleur autour d'un cylindre horizontal lorsque le diamètre du cylindre extérieur devient infini et tendent vers le cas du transfert de chaleur quasi-stationnaire à un fluide situé à l'intérieur d'un cylindre horizontal lorsque le diamètre du cylindre intérieur tend vers zéro. L'étude s'applique à des géométries comprenant des cylindres horizontaux concentriques ou excentrés et à des rangées de cylindres à l'intérieur de l'enceinte cylindrique.

## KORRELATIONSGLEICHUNGEN FÜR DEN WÄRMEÜBERGANG BEI NATÜRLICHER KONVEKTION ZWISCHEN HORIZONTALEN ZYLINDERN

**Zusammenfassung**—Unter Verwendung eines Leitung-Grenzschichtmodells werden Korrelationsgleichungen für den Wärmeübergang bei natürlicher Konvektion an horizontalen Zylindern mit zylindrischer Ummantelung hergeleitet. Die Korrelation ist gültig sowohl für den Fall der Wärmeleitung wie für den Fall laminarer und turbulenter Strömung. Für den Fall, daß der äußere Zylinder unendlich ausgedehnt wird, nähern sich Ergebnisse denen für den Wärmeübergang an einem freien horizontalen Zylinder; geht der Durchmesser des inneren Zylinders gegen Null, so erhält man die Werte für den quasi-stationären Fall des Wärmeübergangs an ein Fluid in einem horizontalen Zylinder. Die Korrelation umfaßt die konzentrische und exzentrische Anordnung sowie die Anordnung mehrerer Zylinder in einem äußeren Zylinder.

## КОРРЕЛЯЦИОННЫЕ УРАВНЕНИЯ ДЛЯ ПЕРЕНОСА ТЕПЛА ЕСТЕСТВЕННОЙ КОНВЕКЦИЕЙ МЕЖДУ ГОРИЗОНТАЛЬНЫМИ КРУГЛЫМИ ЦИЛИНДРАМИ

**Аннотация** — Получены корреляционные уравнения для переноса тепла естественной конвекцией от горизонтальных цилиндров в пространство между цилиндрами. Для вывода уравнений использована модель теплопроводности в приближении пограничного слоя. Уравнения справедливы для описания процесса теплопереноса за счет теплопроводности, а также конвективного переноса ламинарного и турбулентного течений. Полученные результаты сходны с результатами по переносу тепла от горизонтального цилиндра, когда диаметр наружного цилиндра имеет бесконечно большое значение, и по квазистационарному переносу тепла к жидкости внутри горизонтального цилиндра, когда диаметр внутреннего цилиндра приближается к нулю. Уравнения представлены для горизонтальных, концентрических и эксцентрических геометрий, а также пучков цилиндров, помещенных внутри внешнего цилиндра.